Gauss law infinite sheet

from the Office of Academic Technologies on Vimeo. Example 5- Electric field of an infinite sheet of charge Now let's consider an example of infinite sheets of charge with the surface charge density σ coulombs per measure squared. In this case, we have a loaded plate and it is very large, goes to plus infinity in both dimensions and minus infinity, let's say, in these dimensions. Let's assume it is positively charged and surface charge density is given as σ coulombs per meter square. We want to calculate the electric field that it generates at a certain distance from this distribution. Let's assume our point of interest is somewhere over here. In order to solve this problem, we shall apply Gauss law, which is E dot dA integrated over a closed surface, s, is equal to net charge enclosed in the volume surrounded by this hypothetical closed surface divided by ε0. We will choose, in this case, our closed surface as a cylindrical surface passing through the charging sheet. Something like this. Therefore, it goes through the sheet of charge through this region and extends to the other side of the sheet. Since this cylindrical surface looks like a pillbox, this method is also known as pillbox method. The electric field generated by such a wide charge sheet will come from the sheet and extend to infinity on both sides. Therefore, on the right side, they will point to the right. On the left side, they will point to the left, extending to infinity. When we look at our pill box, which is a cylinder, it is made of the circular sides on both sides and also a rectangular surface that wraps around these circular regions. In that sense, if you consider our surface over here on the right side of the plate relative to the circular side, the electric field will point to the right in this direction, and the surface vector will be perpendicular to the surface. Therefore, for this surface over here, it will be perpendicular to it and will point in the same direction with the electric field vector. For the side surface, however, which is this surface over here, the associated surface vector will be perpendicular to this surface. Here, for example, it will point perpendicular to the point dA and the electric field vector right at that point, again, will point in the right direction relative to the surface. Therefore, the angle between these two vectors for the side surface will be 90 degrees. If you go back to the other side of the plate, now for the side surface the electric field is, again, pointing to the left in this case. This surface area vector is perpendicular to the surface, to the side surface dA. Therefore the angle between these two vectors will be only 90 degrees. And if you consider this side of the cylindrical surface, the electric field at that point points to the left and the corresponding area vector over here also be in the same direction of this vector as it is perpendicular to the surface. Now, by applying Gauss law and given this cylindrical surface, pillbox, we can divide the pillbox into its surfaces that eventually make the entire pillbox. For the right side, let's number these surfaces as surface 1, surface 2 and the other side, surface 3 and surface 4. So we can separate the entire closed surface integral of E point dA in the sum of open surfaces integrals that eventually make the entire cylindrical surface. Therefore, integral over the first surface, which is the circular surface, and for that surface, if we express E dot dA in explicit form, we have E-magnitude, dA-magnitude, times the cosine of the angle between them, which is the cosine of 0. Now, when we chose this pillbox, we chose it so that one of these circular sides passes through the point of interest, and like the entire pillbox, the entire cylindrical surface, passes through the plate. So this is the integral one that is taken over one of these circular pages here. Now, let's take the integral over the second surface, which is the side surface of the cylinder on the right side of the plate, and for that region we have E magnitude dA magnitude times cosine of the angle between these two vectors, and for that matter it is 90 degrees. Move on, plus integral over the third surface, and the third surface is the side surface of the cylinder on the other side of this plate. For that region, again, we have E magnitude dA magnitude times cosine of the angle between these two vectors and it is also 90 degrees. Plus the integral of the fourth surface, and there is other circular surface that is on the left side of this plate, and for that region we have E magnitude dA magnitude times, again, the angle between E and dA for that region is 0 degrees. It should be equal to q-enclosed above ε0. Since the cosine of 90 is 0, integrals over the second surface and the third surface will not contribute to our closed surface integration. Only contributions will come from the integrals during the first and fourth, or over circular surfaces of this pillbox of the cylinder. Cosine of 0 is 1, and when we look at our chart, when we are on these circular surfaces, we will be the same distance from the charge, from the distribution. Therefore, the extent of the electric field vector will be the same for these regions. Over these surfaces, the electric field is constant; we can take it outside the integral, and these are identical integrals. Therefore, we end up with E-integral over the first surface of dA, plus E-times integral over the fourth surface of the cylinder should be equal to q-enclosed over ε0. This is the hypothetical Gaussian surface of our choice. Therefore, we can assign or we can select any cross-sectional area that we want. Let's say A be the cross-sectional area of the pill box of our choice. If so, if we integrate dA, the incremental element, all over this first surface, eventually we will end up with the area of that surface and it will give us only A. In the same way, for the integral over the fourth surface, if we add all these dA is to each other throughout the circular surface 4, we will end up with the area of the total surface area of that surface. So the left side will eventually give us 2 times E times A, which is equal to q-enclosed over ε0. q-enclosed, again, by definition, is the net charge inside of the region surrounded by Gaussian surfaces. Gaussian surface is the co-surface of this pill box that encloses only this shaded area of the charge distribution. Therefore, whatever the fee that we are interested is, the amount of charge along this region, along this area. We get the surface charge density of the distribution. Therefore, we can easily calculate the q-enclosed knowing this charge density. Since this area is equal to the cross-sectional surface of this cylinder and we said we chose this cylinder with a cross-sectional area of A, then we can express q-enclosed as the surface charge density σ, which is coulombs per meter square, times the area of the region that we are interested with, and that is A. When we express q-enclosed, in terms of surface charge density , when our expression becomes 2EA is equal to σA over ε0. Dividing both sides with cross-sectional area A, we can eliminate A on both sides and solve for the electric field, the extent of the electric field generated by this infinite plate or sheet of charge, E equals σ over 2ε0. An interesting thing in this result is that σ is constant and 2ε0 is constant. The electrical field from such a charge distribution is equal to a constant and it is equal to the surface charge density divided by 2ε0. Of course, infinite sheet metal of charge is a relative concept. Let us recall the electric field of emissions distribution as we did in the past by applying Coulomb's law. Recall discharge distribution. In that case, we considered a disc charge that was evenly charged throughout its surface with a radius R, and we calculate the electric field along its axis z distance from the center. We have found that the electric field pointed along the axis in an upward direction. The charge had the disc at positive charge of Q coulombs and we obtained a result for the electric field magnitude that E equals Q over 2πε0R2 times, in parentheses, 1 minus z over z2 plus R2 in square root. Now, if we look at this distribution for a specific case such that we approach along the axis towards the center of the disc, in other words for points so that z is much, much smaller than the radius of the disc, then naturally R, because z is much, much smaller than R, z over R will be much, much less than 1. To take advantage of this condition, if we rewrite the expression so that by taking R outside the square root mount, then we will have z in the numerator, R2 will come R outside, and inside of the square root we will have z2 over R2 and plus 1, closed parentheses. Since z over R is much, much less than 1, then z2 over R2 will be even less than 1, which can be neglected, therefore in comparing to 1. Then the square root, we will only end up with 1. Here too we have z over R, which will be also very, very small in comparing to this one. Then we can neglect it in comparison to it too. So we can end up with an approximate expression of the electric field that will be equal to this quantity. But if we look at over here, π times R2 will give us the area of this disc, and then we will have charge per unit area, and that's nothing but the surface load density of this disc. Since Q over πR2 is equal to σ, then the expression reduces into a form of σ over 2ε0. You can easily see now this is the identical result that we received from the endless sheet fee distribution. It indicates that, when we approached the disc so close that our point of interest, the interests of distance along the axis are much, much smaller than the radius of the disc, then we will complete this distribution as an infinite sheet charge. For these points, the electric field it generates will be a constant and it will be equal to the surface charge density divided by 2 times the admissibility of free space. Space.